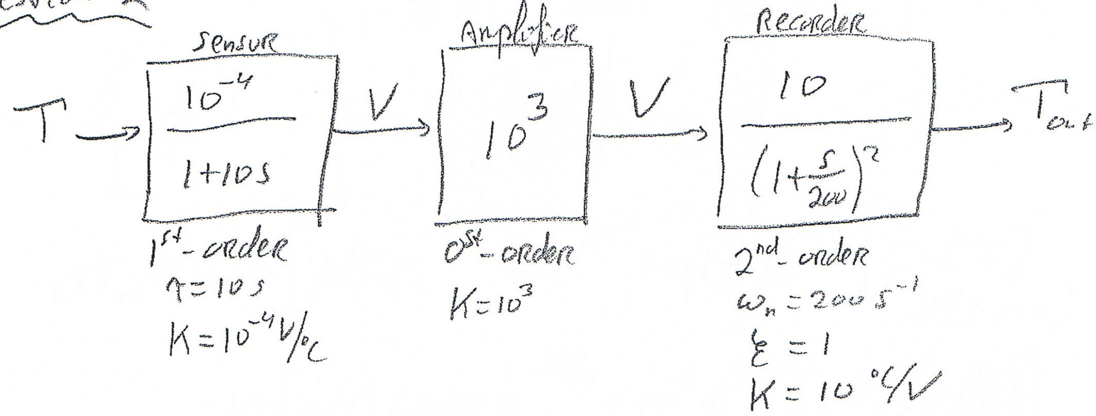
Midterm exam PMS, 05/10/2011

Question 2



a)

$$\frac{\Delta \tilde{T}_{out}}{\Delta \tilde{T}_{in}} = G(s) = \frac{1}{(1+10s)} \cdot \frac{1}{(1+\frac{s}{200})^2}$$

with $\Delta \tilde{T} = \frac{10}{s}$ [°C] $\Rightarrow \Delta \tilde{T}_{out} = 10 \frac{1}{s} \cdot \frac{1}{(1+10s)} \frac{1}{(1+\frac{s}{200})^2}$

$$= 10\left[\frac{A}{s} + \frac{B}{(1+10s)} + \frac{C}{(1+\frac{s}{200})} + \frac{D}{(1+\frac{s}{200})^2}\right]$$

find A, B, C, D:

$$A(1+10s)(1+\tfrac{s}{200})^2 + Bs(1+\tfrac{s}{200})^2 + Cs(1+10s)(1+\tfrac{s}{200}) + Ds(1+10s) = 1$$

$s=0$: $A = 1$

$s = -1/10$: $B(-\frac{1}{10})(1-\frac{1}{2000})^2 = 1 \Rightarrow B = -10{,}01$

$s = -200$: $D \cdot (-200)(1-2000) = 1 \Rightarrow D = 2{,}5 \cdot 10^{-6}$

$s = \frac{1}{10}$: $A \cdot 2 \cdot (1+\frac{1}{2000})^2 + B\frac{1}{10}(1+\frac{1}{2000})^2 + C\frac{1}{10} \cdot 2 \cdot (1+\frac{1}{2000})$

$+ D\frac{1}{10} 2 = 1$

$\Rightarrow$ $C = 5 \cdot 10^{-6}$

$\Delta \tilde{T}_{out} \approx 10\left[\frac{1}{s} - \frac{10.01}{(1+10s)} + \frac{5 \cdot 10^{-6}}{(1+\frac{s}{...})} + \frac{2{,}5 \cdot 10^{-6}}{(...)^2}\right]$

$$\Rightarrow \Delta \tilde{T}_{out} = 10\left[\frac{1}{s} - \frac{1{,}001}{\frac{1}{10}+s} + \frac{10^{-3}}{200+s} + \frac{0{,}1}{(200+s)^2}\right]$$

$\mathcal{L}^{-1}$

$$\Delta T_{out}(t) = 10\left[\mu(t) - e^{-t/10} + 10^{-3}e^{-200t} + 0{,}1\,t\,e^{-200t}\right] \quad [^\circ C] \quad (t \text{ in } s)$$

$$= 10\left[\mu(t) - 1{,}001\,e^{-t/10} + 10^{-3}e^{-200t}(1+100t)\right]$$

b) the bandwith is basically determined by first element (sensor). The last element starts kicking in at $\omega \simeq \omega_n$ for which the first element does not respond any more:

$\omega \ll \omega_n$:

$$G(s) = \frac{1}{1+10s}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+100\omega^2}} = \frac{1}{\sqrt{2}} \quad \text{(def. of bandw.)}$$

$$\hookrightarrow \underline{\underline{\omega_b = 0.1\ s^{-1}}}$$

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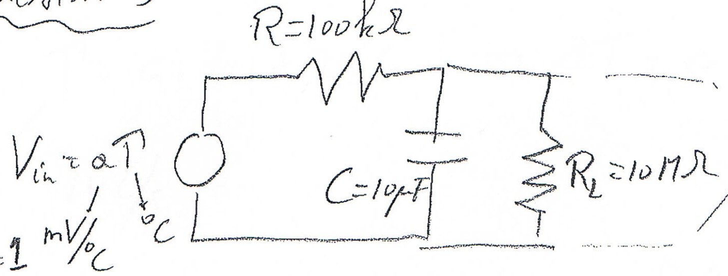
Question 3



$\alpha = 1\ mV/°C$

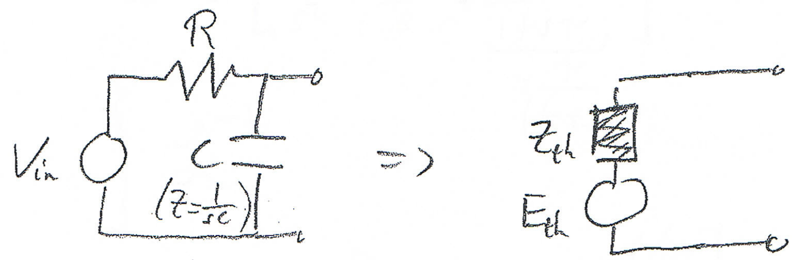
$V_{out} \to T_{out} = V_{out}/\alpha$

a) steady-state sensitivity => no current through C (no time dependence)

$$V_{out} = V_{in}\frac{R_L}{R+R_L} = V_{in}\frac{10^7\Omega}{10^5\Omega+10^7\Omega} = V_{in}\frac{100}{1+100} = 0{,}99\,V_{in}$$

$$\Rightarrow T_{out} = 0{,}99\,T = \underline{\underline{19{,}8\,°C}}$$

b)



$$E_{th} = \Delta\tilde{V}_{in}\frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \underline{\underline{\frac{\Delta\tilde{V}_{in}}{1+sRC}}}\ ;\quad Z_{th}:\ C//R:\ \frac{1}{Z_{th}} = sC + \frac{1}{R}$$

$$\Rightarrow \underline{\underline{Z_{th} = \frac{R}{1+sRC}}}$$

c)
$$\Delta\tilde{V}_{out} = E_{th}\frac{R_L}{Z_{th}+R_L} = \frac{\Delta\tilde{V}_{in}}{(1+sRC)}\frac{R_L}{\left(\frac{R}{1+sRC}+R_L\right)} = \frac{\Delta\tilde{V}_{in}}{\frac{R}{R_L}+1+sRC}$$

$$\frac{\Delta\tilde{T}_{out}}{\Delta\tilde{T}_{in}} = \frac{\Delta\tilde{V}_{out}}{\Delta\tilde{V}_{in}} = G(s) = \frac{1}{1+s\tau}\ \text{ with } \tau = RC = 1s$$

$$\simeq \frac{\Delta\tilde{V}_{in}}{1+s\underbrace{RC}}$$

$\tau = 10^5\cdot10^{-5}s = 1s$

) cont$^{\underline{d}}$

$$T = a \cdot t + 20°C \Rightarrow \Delta T = a \cdot t \overset{\mathcal{L}}{\rightsquigarrow} \Delta\tilde{T} = \frac{a}{s^2}$$

$°C$ $\quad 10°C/s \quad s$

$$\Rightarrow \Delta\tilde{T}_{out} = a\frac{1}{s^2}\cdot\frac{1}{(1+s\tau)} = a\left[\frac{A}{s^2} + \frac{B}{s} + \frac{C}{1+s\tau}\right]$$

Get A, B, C: $A(1+s\tau) + Bs(1+s\tau) + Cs^2 = 1$

$\underline{s=0}$: $A=1$; $\underline{s=-1/\tau}$: $C=\tau^2$; $\underline{s=\frac{1}{\tau}}$: $B=-\tau$

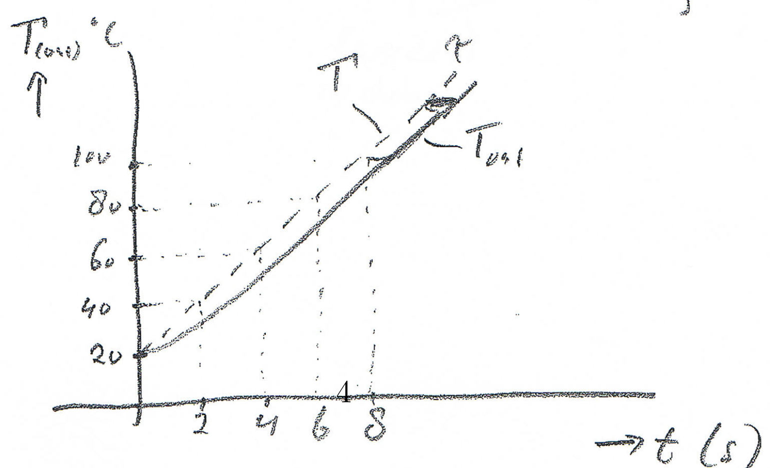
$$\Rightarrow \Delta\tilde{T}_{out} = a\left[\frac{1}{s^2} - \frac{\tau}{s} + \underbrace{\frac{\tau^2}{1+s\tau}}_{=\frac{\tau}{(\frac{1}{\tau}+s)}}\right]$$

$\mathcal{L}^{-1}$

$$\Delta T_{out}(t) = a\left[t - \tau + \tau e^{-t/\tau}\right] = a\left[t - \tau(1 - e^{-t/\tau}\right.$$

$t \gg \tau \Rightarrow \Delta T_{out} \cong a[t-\tau]$

$t \ll \tau \Rightarrow \Delta T_{out} \approx a\left[t - \tau\left(1 - 1 + \frac{t}{\tau} - \frac{t^2}{2\tau^2}\cdots\right)\right] = a\left[t^2/2\tau\right]$



d) $T = 20 + 10\sin(\omega_1 t) + 1 \cdot \sin(\omega_2 t)$ [°C]

$$\omega_1 = \frac{2\pi}{2\text{ hours}} = \frac{2\pi}{2 \cdot 3600[s]} = 8{,}7 \cdot 10^{-4}\ [s^{-1}]$$

$$\omega_2 = \frac{2\pi}{0{,}5s} = 12{,}6\ [s^{-1}]$$

$$T_{out} = 20 + |g(j\omega_1)| \sin(\omega_1 t + \mathrm{Arg}(g(j\omega_1))) + |g(j\omega_2)| \sin(\omega_2 t + \mathrm{Arg}(g(j\omega_2)))\ [°C]$$

with $g(j\omega) = \frac{1}{1+j\omega\tau} = \frac{(1-j\omega\tau)}{(1+\omega^2\tau^2)}$

$\Rightarrow |g| = \frac{1}{\sqrt{1+\tau^2\omega^2}}$ and $\mathrm{Arg}(g) = \mathrm{atan}(-\omega\tau)$

(with $\tau = 1s$)

$$\Rightarrow T_{out}(t) = 20 + \underbrace{10\sin(\omega_1 t)}_{\substack{(\omega\tau \ll 1) \\ \text{no change}}} + 0{,}079\sin(\omega_2 t \underbrace{-1{,}49}_{\text{close to } -\pi/2})\ [°C]$$

Question 1

a)

from (I) and (III) : $\Delta O = K_I \Delta I_I$ (at $I=0$) : lowest input value

$\Rightarrow 7-6 = K_I (28-25) \Rightarrow \boxed{K_I = \frac{1}{3} \left(\frac{mA}{^\circ C}\right)}$

Thus, temperature is the interfering variable here, since at lowest input value can change the output (book, page 29)!

Now, from (I) and (II) at $I=0$: Since changing the voltage from 10 V to 14 V at fixed $I_I = 25^\circ C$ does not change the output, the environmental variable "voltage" is not an interfering input.

From (I) and (II): a change of voltage from 10V to 14V at $\underbrace{I=4}_{\text{mid-range input}}$ results in a change in the output from 12.4 to 15.6.

Thus, "voltage" must be a modifying input I_M, and from [2.25] we will have:

$$\boxed{K_M = \frac{1}{I} \frac{\Delta O}{\Delta I_M} = \frac{1}{4} \frac{15.6-12.4}{14-10} = 0.2 \left(\frac{mA}{V\,bar}\right)}$$

$$\Rightarrow O = (K + 0.2 I_M) I + a + \frac{1}{3} I_I$$

Considering (I) $\rightarrow I_I = 0$ (difference with standard situation) for $I = 0$ $\Big\} \Rightarrow O = a \Rightarrow \boxed{a = 6 \ (mA)}$

Hence, $O = (K + 0.2 I_M) I + 6 + \frac{1}{3} I_I$

Considering (I) $\rightarrow I_I = 0,\ I_M = 0$ for $I = 10$ bars $\Big\} \Rightarrow O = K \times 10 + 6$

$$\Rightarrow \boxed{K = \frac{22-6}{10} = 1.6 \left(\frac{mA}{bar}\right)}$$